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Testing of conservation laws for the interaction of plasma waves with difference-frequency whistlers in the solar corona

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The conservation laws for nonlinear interaction of plasma waves with difference-frequency $\omega^{t-} = \omega^l - \omega^w$ whistlers are analyzed. It is shown that this process in the solar corona, like sum-frequency coalescence, is also efficient and is possible for wave vectors $\mathbf{k}^l$ and $\mathbf{k}^w$ with magnitudes of ~ 0.3 cm^{-1} to ~ 0.45 cm^{-1} and almost mutually parallel directions forming large angles $\sim 80°$ relative to the magnetic field. Electromagnetic radiation in the ordinary mode can be emitted at angles close to $\vartheta^t \sim \pm\cos^{-1}2(\omega^w/\omega_H)$. Simultaneous emission at the sum and difference frequencies can induce $\sim 2\omega^w$ frequency splitting relative to the absorption band in fibers with intermediate frequency drift in solar type IV radio bursts.

INTRODUCTION

The interaction of plasma waves (ℓ) with whistlers (w) is usually investigated for quasi-longitudinal propagation of both waves. In this case the laws of conservation of energy and momentum (or phase locking) are satisfied for the emission of an electromagnetic wave (t) at the sum frequency $\omega^t = \omega^\ell + \omega^w$, $\mathbf{k}^t = \mathbf{k}^l + \mathbf{k}^w$, where the wave vectors $\mathbf{k}^\ell$ and $\mathbf{k}^W$ are in opposite directions and have close magnitudes, since $k^t \ll k^\ell$ and $k^t \ll k^W$ (Refs. 1, 10, and 14). The decay process $\ell \to t + w$ at the difference frequency $\omega^{t-} = \omega^\ell - \omega^W$, $\mathbf{k}^t = \mathbf{k}^l - \mathbf{k}^w$ does not usually take place in cosmic plasma under these conditions (Ref. 12, Sec. 6.4).

However, it has long been known from laboratory experiments that low- and high-frequency waves at both the sum and difference frequencies can exchange energy in a plasma.[9] A series of observations of the fine structure of solar type IV radiation in the form of fibers with an intermediate drift frequency (fiber bursts, which are attributable to the coalescence mechanisms $\ell + w \to t$) clearly exhibit emission from the low-frequency edge of a typical fiber, possibly as the result of decay into the difference frequency ω^{t-} (Refs. 14 and 15). These observations encouraged the authors to undertake detailed testing of the conditions for resonance in application to the difference-frequency decay process $\ell \to t + w$. This process can be very significant for any periodic fibers in radiation, because the possibility of splitting of the radiation with a frequency separation of the order of twice the whistler frequency must then be taken into account.

ESTIMATION OF THE OPTICAL THICKNESS OF THE PROCESS $\ell \to t + w$

According to Tsytovich (Ref. 11, Sec. V.13) and Melrose,[6,7] the interaction of high- and low-frequency waves can take place either at the sum frequency or at the difference frequency. Coalescence and decay can be equally efficiency processes. The kinetic equations for the rate of growth of transverse waves in the processes $\ell + w \to t$ and $\ell \to t + w$ differ only by the signs in front of the second and third terms (Eq. (B.2) in Ref. 6):

$$\frac{dN_\pm{}^t}{dt} = \int \frac{d^3\mathbf{k}^l}{(2\pi)^3}\int \frac{d^3\mathbf{k}^w}{(2\pi)^3} u_\pm^{tlw}[N^lN^w \mp N^t(N^l \pm N^w)]$$

(the upper signs for coalescence, and the lower signs for decay), where $N^{t,\ell,W}$ are dimensionless wave intensities. The only difference in the equations for the coalescence and decay probabilities $u_\pm^{t\ell W}$ is in the signs of the resonance conditions in the argument of the delta function.

We first of all determine more precisely the efficiency of the decay process in comparison with coalescence, from the point of view of sufficient optical thickness ($\tau > 1$). It has been shown in several papers that the observed radiative flux from fibers is attributable to the coalescence process $\ell + w \to t$ only in an optically thick source, either for $N^W \gg N^\ell$ (Ref. 4) or for the opposite relationship of the wave intensities $N^W \ll N^\ell$ (Refs. 1 and 10). The following equation holds for decay in the notation of Ref. 10 with allowance for the above-mentioned sign changes in the kinetic equation:

$$dN^t/dz = \Gamma_1 + N^t\Gamma_2, \tag{1}$$

where

$$\Gamma_1 = \frac{1}{4}\left(\frac{\omega^w}{\omega_H}\right)^{3/4}\left(\frac{\omega_H}{\omega_p}\right)^2 N^lN^w;\quad \Gamma_2 = \frac{1}{4}\left(\frac{\omega^w}{\omega_H}\right)^{1/4}\left(\frac{w_H}{w_p}\right)^3 N^l - \frac{1}{4}\left(\frac{\omega^w}{\omega_H}\right)^{3/4}\frac{\omega_H}{\omega_p}N^w,$$

$$z = \frac{\omega_p}{c} v^t_{gr} t \frac{\omega_p}{\Delta\omega^w},$$

and ω_H and ω_p are the electron cyclotron and plasma frequencies. It is readily verified that the solution of Eq. (1) differs from the analogous solution for coalescence [10] only by the sign in the argument of the exponential function $N^t = -(\Gamma_1/\Gamma_2)\cdot(1 - \exp \Gamma_2 z)$. Hence it follows that in order for N^t to be positive, Γ_2 must be negative ($\Gamma_2 < 0$), and the condition $N^w > N^l \left(\frac{\omega_H}{\omega^w}\right)^{1/2} \left(\frac{\omega_H}{\omega_p}\right)^2 \approx 2.5 \cdot 10^{-3} N^l$ therefore holds. In order for a source to be optically thick with respect to the decay process $\ell \to t + w$ (i.e., in order to have $|\Gamma_2 z| > 1$), the condition for the nonlinear interaction length $z_{n\ell} > |\Gamma_2|^{-1}$ must be satisfied. For the characteristic values $N^W \sim 10^{-5}$, $N^\ell \sim 10^{-6}$, $\omega^W/\omega_H = 0.2$, and $\omega_p/\omega_H = 30$ we have $z_{n\ell} \approx 0.5 \cdot 10^8$ cm. This quantity is of precisely the same order as the dimensions of the fiber source (the length of the whistler wave packet) (Ref. 14).

With the foregoing considerations in mind, we can state that coalescence and decay at the frequencies $\omega^{t\pm} = \omega^\ell \pm \omega^W$ can be equally efficient in an optically thick source, and the maximum electromagnetic radiation intensity is given by the expression $N^t \lesssim \Gamma_1/\Gamma_2$ or, in correspondence with the notation and signs,

$$N^t \leqslant \frac{N^l N^w}{\sqrt{\frac{\omega_H}{\omega^w}\frac{w_H}{\omega_p}} N^l \pm \frac{\omega_p}{\omega_H} N^w}.$$

TESTING OF THE CONSERVATION LAWS

It has been shown[15] that the conditions for the emission of radiation from the coronal plasma at the frequency ω^{t-} at the typical values $\omega_p/\omega_H \approx 30$ and $T_e \sim 10^{-6}$ K place a lower bound on the magnitude of the plasma wave vector $k \gtrsim 0.3$ cm^{-1}. On the other hand, satisfaction of the conditions $k^t \ll k^\ell$ and $k^t \ll k^W$ implies (as in the case of the coalescence process) that $k^\ell \sim k^W$. We obtain the lower bound $k^W \gtrsim 0.3$ cm^{-1}. According to the whistler dispersion relation, wave vectors of such large magnitude must be directed at large angles relative to the magnetic field, $\vartheta^W \sim 70$–$80°$. It follows from the condition $k^t \ll k^\ell$, k^W that the angle between k^ℓ and the magnetic field vector must also be of the same order, $\vartheta^t \sim 70$–$80°$. The simple interaction scheme of the wave vectors $\mathbf{k}^\ell$ and $\mathbf{k}^W$ shows that they must be in the same direction with a very small angle between them, $\vartheta \sim 1°$ (Ref. 15).

We now attempt to determine the ranges of possible angles ϑ^ℓ and ϑ^t (ϑ^t is the angle between $\mathbf{k}^t$ and $\mathbf{H}$) and the relative values of the wave numbers k^ℓ/k^W wherein resonance relations hold for interaction at the difference frequency $\omega^{t-} = \omega^\ell - \omega^W$. It is evident from the dispersion curves that only an ordinary wave can participate in such an interaction. We use simple dispersion relations for whistlers, plasma, and ordinary electromagnetic waves, but with allowance for the magnetic field, since the problem at hand is to determine the angles between $\mathbf{k}^t$, $\mathbf{k}^\ell$, $\mathbf{k}^W$, and $\mathbf{H}$:

$$w^w = \frac{\omega_H k^{w^2} c^2 \cos\vartheta^w}{\omega_p^2 + k^{w^2} c^2},$$

$$\omega^{l^2} = \omega_p^2 + 3k^{l^2} v_T^2 + \omega_H^2 \sin^2\vartheta^l,$$

$$\omega^{t^2} = k^{t^2} c^2 + \omega_p^2 \left(1 + \frac{\omega_H}{\omega^t}\cos\vartheta^t\right)^{-1}.$$

We simplify the last two equations using expansions subject to the condition that their second and third terms are small and $\omega_H/\omega_p \ll 1$:

$$\omega^l \approx \omega_p \left(1 + \frac{3}{2}\frac{k^{l^2} v_T^2}{\omega_p^2} + \frac{\omega_H^2 \sin^2\vartheta^l}{2\omega_p^2}\right),$$

$$\omega^t \approx \omega_p \left(1 + \frac{k^{t^2} c^2}{2\omega_p^2} - \frac{\omega_H}{2\omega_p}\cos\vartheta^t\right).$$

We substitute these expressions in the first resonance condition $\omega^{t-} = \omega^\ell - \omega^W$, determine k^{t2} from the result, and substitute it in the relation for the wave numbers:

$$k^{t^2} = \frac{3k^{l^2} v_T^2}{c^2} + \frac{\omega_H^2 \sin^2\vartheta^l}{c^2} - 2\omega_p \frac{k^{w^2}\omega_H \cos\vartheta^w}{\omega_p^2 + k^{w^2} c^2} + \frac{\omega_p \omega_H}{c^2}\cos\vartheta^t = k^{l^2} + k^{w^2} - 2k^l k^w \cos\vartheta. \quad (2)$$

Using this procedure to eliminate k^t, we estimate $\cos\vartheta^t$:

$$\cos\vartheta^t = \frac{k^{w^2} c^2}{\omega_p \omega_H}\left[\left(\frac{k^l}{k^w}\right)^2 (1 - 3\beta_T^2) - 2\frac{k^l}{k^w}\cos\vartheta + 1\right] + 2\frac{\omega^w}{\omega_H} - \frac{\omega_H}{\omega_p}\sin^2\vartheta^l. \quad (3)$$

Here $\beta_T = v_T/c$, and the ratio k^ℓ/k^W is a parameter. For $\omega_H/\omega_p \sim 1/30$ the quantity $\cos\vartheta^t$ is determined maintly by the second term in Eq. (3), $2(\omega^W/\omega_H)$, i.e.,

$$\vartheta^t \sim \pm \arccos 2\frac{\omega^w}{\omega_H}. \quad (4)$$

The angle ϑ^t is somewhat larger or smaller than this value, depending on the sign of the bracketed term in Eq. (3); the sign, in turn, is governed mainly by the ratio k^ℓ/k^W, which is close to unity. The sign in Eq. (4) must also be chosen according to the ratio k^ℓ/k^W.

The bracketed term in Eq. (3) changes sign when

$$\left(\frac{k^l}{k^w}\right)^2 (1 - 3\beta_T^2) - 2\left(\frac{k^l}{k^w}\right)\cos\vartheta + 1 = 0,$$

so that

$$\frac{k^l}{k^w} = \frac{\cos\vartheta \pm \sqrt{\cos^2\vartheta - (1 - 3\beta_T^2)}}{1 - 3\beta_T^2}. \quad (5)$$

In order for k^ℓ/k^W to be real-valued, it is necessary that $\cos^2\vartheta - (1 - 2\beta_T^2) > 0$, i.e., $\cos\vartheta > \pm\sqrt{1 - 3\beta_T^2}$, so that the upper bound of the angle $\vartheta < 1.28°$ is obtained at the coronal temperature $T_e \sim 10^6$ K, or for $v_T \approx 3.89 \cdot 10^8$ cm/s. We then obtain the two values $k^\ell/k^W = 1.00235$ and 0.99816 from Eq. (5), i.e., the bracketed term in Eq. (3) changes sign when the ratio k^ℓ/k^W passes through unity. Substituting these values in Eq. (3) and taking $k^W = 0.4$ cm^{-1}, $\omega_p/2\pi = 3 \cdot 10^8$ Hz, $\omega_H/\omega_p = 1/30$, and $\vartheta = 1°$, we obtain the maximum possible angles $\vartheta^t \lesssim 84°$. Very close wave numbers k^ℓ and k^W are excluded, because they give values of k^ℓ that are too small (the angle ϑ is very small).

We estimate the external values of k^t and, hence, of the ratio k^ℓ/k^W from the equation $\cos^2\vartheta - (1 - 3\beta_T^2) > 0$, i.e., $\cos\vartheta > \pm\sqrt{1 - 3\beta_T^2}$. In order for k^ℓ to be real-valued, it is necessary that $k^t > \pm k^W \sin\vartheta$. For example, if $k^W = 0.4$ cm^{-1} and $\vartheta = 1°$, we have $k^t \gtrsim 7 \cdot 10^{-3}$ cm^{-1}. According to the dispersion re-

For example, if $k^w = 0.4$ cm^{-1} and $\vartheta = 1°$, we have $k^t \gtrsim 7\cdot10^{-3}$ cm^{-1}. According to the dispersion relation for ω^t, the maximum value of k^t is $\sim1.2\cdot10^{-2}$ cm^{-1}. The maximum and minimum ratios k^ℓ/k^w corresponding to these extremal values of k^t are then ~1.025 and ~0.975. Substituting the latter in Eq. (3) and retaining the same values of the other parameters as before, we obtain the minimum value of the angles $\vartheta^t \gtrsim 28°$.

Angles ϑ^t close to 0 and 180° are excluded, because ϑ^t and ϑ^w cannot be close to 90°, and the ratio k^ℓ/k^w is close to unity. The quantity $\cos\vartheta^t$ depends weakly on ϑ^ℓ and ϑ in Eq. (3).

The upper bound of the wave numbers k^ℓ and k^w depends on the increase in Landau damping for ℓ-waves. It therefore follows that the upper bound is $k^\ell \sim 0.48$ cm^{-1}, i.e., an order of magnitude lower than the Debye number $k_d = \omega_p/v_T \approx 4.8$ cm^{-1}. The lower bound of k^ℓ and k^w is determined by the minimum frequencies ω^{t-} of radiation emanating freely from the source. For example, at the minimum frequency $\omega^{t-} = \omega_p + 0.05\,\omega_H$ for $\omega^w/\omega_H \sim 0.1$ we obtain the minimum values k^ℓ, $k^w \approx 0.21$ cm^{-1}.

CONCLUSION

We have determined that the decay process $\ell \to t + w$ at the difference frequency $\omega^{t-} = \omega^\ell - \omega^w$ takes place for large angles ϑ^w, $\vartheta^\ell \sim 80°$ and almost parallel vectors $\mathbf{k}^\ell$ and $\mathbf{k}^w$ with magnitudes from ~0.3 cm^{-1} to ~0.45 cm^{-1}. Electromagnetic radiation in the ordinary mode can emerge at angles $\vartheta^t \sim \pm\cos^{-1}2(\omega^w/\omega_H)$ relative to the magnetic field, i.e., within the range $28° \lesssim \vartheta^t \lesssim 84°$.

Landau damping is known to inhibit the propagation of whistlers at large angles. The principal evidence for this fact lies in observations of magnetospheric whistler trajectories with multiple reflections. Moreover, according to Hashimoto and Kimura,[13] Landau damping has a maximum at intermediate angles $\vartheta^w \sim 30$-$50°$, and decreases with a further increase in ϑ^w. This is explained by the fact that the complete expression for the decay factor for Landau damping includes a linear combination of several exponential terms with different signs. Mal'tseva and Chernov[3] have carried out more complete calculations of the damping of whistlers in the solar corona plasma over the entire range of existence of whistlers and for arbitrary angles relative to the magnetic field. An intricate combination of Landau damping and cyclotron damping at different frequencies is observed, and, in particular, the conclusion of Hashimoto and Kimura[13] as to weak Landau damping at large angles ϑ^w is confirmed.

Whistlers at large angles with the magnetic field can be excited by the anomalous Doppler effect. The first calculations of oblique whistlers can be found in Yip's work.[8] Mal'tseva and Chernov[15] have calculated the kinetic instability of oblique whistlers for a large set of parameters of electron beams with conical and temperature anisotropy.

It is also known that whistlers are rendered isotropic in scattering by cold electrons[12] and by low-frequency ion–acoustic waves.[2] Consequently, conditions for the decay process $\ell \to t + w$ must always be met in the long-range propagation of whistlers in the corona. For whistlers excited along the magnetic field, decay can be initiated by reflections in the vicinity of the lower hybrid resonance.

Simultaneous emission at the frequencies $\omega^{t\pm} = \omega^\ell \pm \omega^w$ implies frequency splitting with a separation $\sim2\omega^w$, whose reception can be prevented only by a difference in the directions of emission. Melrose[7] has shown that four-plasmon coalescence and decay processes in an optically thick source at the combination frequencies $\omega^t = \omega^\ell \pm \omega^w \pm \omega^{w'}$ are also possible in the corona.

Thus, many radio phenomena in solar type IV bursts may be associated with difference-frequency decay processes.

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