DOI 10.1007/s11141-017-9826-7
Radiophysics and Quantum Electronics, Vol. 60, No. 7, December, 2017
(Russian Original Vol. 60, No. 7, July, 2017)

GEOMAGNETIC PERTURBATIONS CAUSED BY A SPHERE MOVING IN THE CONDUCTING LIQUID

V. V. Surkov,[1,2] **V. M. Sorokin,**[2*] **and**
A. K. Yashchenko[2]

UDC 537.613

We study the external magnetic-field perturbations caused by flows of a conducting incompressible liquid which streamlines a dielectric sphere moving in it. An analytical solution of the problem is obtained for the case of a potential flow of liquid and an arbitrary orientation of the external magnetic field. Angular distribution of the magnetic and electric field perturbations, as well as the dependence of their amplitudes on distance, are examined. The directions in which the magnetic and electric components of the perturbations are the maximum are determined. Temporal dependences and the spectra of electromagnetic signals are analyzed for different parameters of the problem. The results of analytical research are illustrated by numerical calculations.

1. INTRODUCTION

The motion of surface and underwater vessels in sea water can lead to small local perturbations of the Earth's geomagnetic field. One of the reasons for this phenomenon is the effect of the ship's own magnetic field, which can, for example, arise during its construction due to the magnetization of its hull or ferromagnetic materials that are part of the ship [1]. Another possible reason is the generation of currents in sea water due to its motion around the hull of the ship [2–4]. The origin of these currents is due to the action of the magnetic force on electric charges in sea water. A similar effect of the geomagnetic field perturbation occurs during wave motions of the sea surface (see, e. g., [5]), in particular, tsunami [6], during oscillation of the Earth's conducting layers in seismic waves, under the action of acoustic and internal gravity waves on the ionospheric conducting layers, etc. [7].

The general nature of the geomagnetic disturbances caused by induction currents in sea water near the ship depends significantly on the distribution of the mass-velocity field of the liquid flowing around its hull. However, at far distances the magnetic perturbations can have a more universal character, and therefore the solutions of the simplified model problems can be used for their study. In this paper, we find an exact solution to the problem of the external magnetic field perturbations caused by a laminar flow of a conducting incompressible fluid that flows around a solid dielectric sphere. Based on this solution, the distribution of magnetic perturbations and their dependence on the distance to the sphere is estimated.

2. MOTION OF A SPHERE ALONG THE MAGNETIC FIELD

Consider a dielectric nonmagnetic sphere of radius R, which moves at a constant speed in the conducting homogeneous fluid located in a uniform magnetic field $\mathbf{B}_0$. We will study the magnetic field perturbations $\mathbf{b}$ caused by the electrical currents generated in the flow of a liquid streamlining the sphere.

* sova@izmiran.ru

[1] National Research Nuclear University (Moscow Engineering Physics Institute); [2] N. V. Pushkov Institute of Terrestrial Magnetism, Ionosphere and Radio-Wave Propagation of the Russian Academy of Sciences (IZMIRAN), Troitsk, Moscow, Russia. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika, Vol. 60, No. 7, pp. 617–626, July 2017. Original article submitted April 27, 2016; accepted June 15, 2017.

We now pass to the reference system in which the sphere is at rest. In this reference system, the distributions of the mass flow velocity $\mathbf{V}$ and of the magnetic $\bar{\mathbf{b}}$ and electric $\bar{\mathbf{E}}$ perturbations are stationary, i. e., are time independent. In addition, the magnetic perturbations are small compared with the unperturbed field, $\bar{b} \ll B_0$. Hereafter, for convenience, we omit an overbar in the expression for magnetic perturbations. Then the Maxwell equations in this reference system take the form

$$\mathrm{rot}\mathbf{b} = \mu_0\sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}_0); \qquad \mathrm{rot}\mathbf{E} = 0; \qquad \mathrm{div}\mathbf{b} = 0,$$

where μ_0 is the magnetic constant. Assuming that the conductance coefficient σ of the liquid is constant, we take a curl from both sides of the first equation and, using the second and the third equations, obtain

$$\mathrm{rotrot}\mathbf{b} = -\mu_0\sigma\mathrm{rot}[\mathbf{V} \times \mathbf{B}](\mathbf{V} \times \mathbf{B}_0). \tag{1}$$

Assume that the flow of a liquid is laminar and the effect of its viscosity is negligible. The conducting liquid is affected by a hydrodynamic pressure and a magnetic force with the bulk density $\mathbf{j} \times \mathbf{B}$, where $\mathbf{j}$ is the density of the electrical current. In what follows we will show that under the conditions of this problem, the magnetic term in the Euler equation can be neglected as compared with the gradient of the hydrodynamic pressure. Let us introduce the Cartesian coordinate system x, y, z with the z axis directed along the velocity vector of the incident flow of liquid. Using the well-known Euler equation in the problem of a sphere streamlined by an ideal incompressible liquid (see, e. g., [8]), we write an expression for the mass flow velocity:

$$\mathbf{V} = \frac{R^3}{2r^3}[3\mathbf{e}_r(\mathbf{V}_0\mathbf{e}_r) - \mathbf{V}_0] - \mathbf{V}_0. \tag{2}$$

Here, r and $\mathbf{e}_r$ denote the absolute value and the unit vector of the radius vector which we draw from the sphere center, respectively, and $\mathbf{V}_0$ is the velocity of the incident flow of liquid at infinity. For solving the problem, it is convenient to use a spherical coordinate system r, φ, θ with the polar angle θ reckoned from the direction of the vector $\mathbf{V}_0$, as is shown in Fig. 1.

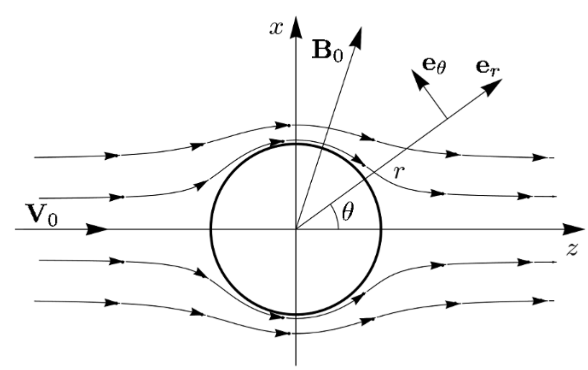

Fig. 1. The coordinate system used for calculation of the perturbation of the magnetic field by a moving sphere.

First we assume that the induction vector of an external magnetic field is parallel to the z axis (i. e.,. the sphere velocity $\mathbf{V}_0$). Since the problem is axially symmetric, all the quantities are independent of the azimuthal angle φ. Substitute the velocity from relation (2) to Eq. (1). Transforming the obtained equation, we write it in components in the form

$$\begin{aligned} &r^2[\hat{T}_1 b_r - 2b_r + \hat{T}_2(\partial_\theta b_r - 2b_\theta)] = K(3\cos^2\theta - 1),\\ &r^2[\hat{T}_1 b_\theta + \hat{T}_2\partial_\theta b_\theta + 2\partial_\theta b_r - \sin^{-2}\theta b_\theta] = K\sin(2\theta), \end{aligned} \tag{3}$$

where we introduced the following designations:

$$\partial_r = \partial/\partial r, \qquad \partial_\theta = \partial/\partial\theta, \qquad \hat{T}_1 = \partial_r(r^2\partial_r), \qquad \hat{T}_2 = \sin^{-1}\theta\partial_\theta(\sin\theta), \qquad K = 3\mu_0\sigma V_0 B_0 R^3/2.$$

We write the equation $\mathrm{div}\mathbf{b} = 0$ in the form

$$r\partial_r b_r + 2b_r + \hat{T}_2 b_\theta = 0. \tag{4}$$

The solution of Eqs. (3) and (4) for the magnetic-disturbance components can be sought in the form of series expansions over associated Legendre functions $P_n^m(\cos\theta)$. However, taking into account that inhomogeneities on the right-hand sides of these equations can be expressed in terms of $P_2^1 = 3\sin\theta\cos\theta$ and $P_2^0 = (3\cos^2\theta - 1)/2$, it can be assumed that $b_\theta \propto P_2^1(\cos\theta)$ and $b_r \propto P_2^0(\cos\theta)$. This angular dependence of the magnetic-perturbation components permits one to seek a solution in the form

$$b_r = f_r(r)\{3\cos^2\theta - 1\}, \qquad b_\theta = f_\theta(r)\sin\theta\cos\theta. \tag{5}$$

Substituting relations (5) into Eqs. (3) and (4), we arrive at the following system of equations for the desired functions f_r and f_θ:

$$r^2 f_r'' + 2rf_r' - 8f_r - 2f_\theta = K/r^2, \qquad r^2 f_\theta'' + 2rf_\theta' - 12f_r - 6f_\theta = 2K/r^2, \qquad rf_r' + 2f_r + f_\theta = 0, \tag{6}$$

where the primes denote a derivative with respect to r. From system (6) we obtain

$$r^4 f_r'' + 4r^3 f_r' - 4r^2 f_r = K. \tag{7}$$

Let us find a general solution to Eq. (7) for the function f_r. Then, substituting f_r into the last equation of system (6), we determine f_θ. As a result, we obtain

$$f_r = C_1/r^4 + C_2 r - K/(6r^2), \qquad f_\theta = 2C_1/r^4 - 3C_2 r, \tag{8}$$

where C_1 and C_2 are indefinite constants. Relations (8) give a general solution to the system of equations (6). Taking into account that magnetic perturbations (8) should be finite at $r \to \infty$, we find that $C_2 = 0$.

Since the sphere is nonconducting and nonmagnetic, the region inside the sphere ($r < R$) is described by Eqs. (3), in which one should put $K = 0$. Solving these equations in a similar way, we find that

$$f_r = C_3 r + C_4/r^4, \qquad f_\theta = 2C_4/r^4 - 3C_3 r. \tag{9}$$

From the condition for the finiteness of the solution at $r = 0$ it follows that $C_4 = 0$. The constants C_1 and C_3 can be found from the condition for the continuity of f_r and f_θ at the sphere boundary, i. e., by equating solutions (8) and (9) at $r = R$. Substituting these solutions into Eq. (5), we obtain the final form of the solution for $r > R$:

$$b_r = \lambda B_0 \frac{R^2}{4r^2}\left(1 - \frac{3R^2}{5r^2}\right)(1 - 3\cos^2\theta), \qquad b_\theta = \lambda B_0 \frac{3R^4}{10r^4}\sin\theta\cos\theta, \qquad \lambda = \mu_0\sigma V_0 R. \tag{10}$$

Consider how the boundary conditions on the sphere surface affect the character of the magnetic perturbations. Let the sphere be covered by a conducting shell, the conductivity of which is much greater than the conductivity of the environment. In this case, it can be assumed that surface currents are generated on the sphere, resulting in that the magnetic field inside the sphere vanishes. Then the tangent component of magnetic perturbations on the sphere surface will undergo a jump, while the normal component should be equal to zero. The boundary condition $f_r(R) = 0$ permits one to find the indefinite constant C_1: $C_1 = KR^2/6$. The solution of the problem for this case has the form ($r > R$)

$$b_r = \lambda B_0 \frac{R^2}{4r^2}\left(1 - \frac{R^2}{r^2}\right)(1 - 3\cos^2\theta), \qquad b_\theta = \lambda B_0 \frac{R^4}{2r^4}\sin\theta\cos\theta. \tag{11}$$

Comparison of solutions (10) and (11) for two considered cases shows that the angular and radial dependences of magnetic perturbations are identical, and their amplitudes are close in magnitude. This trend persists for other cases, as well. It can therefore be concluded that the behavior of the magnetic perturbation (at least, at far distances) depends only weakly on the boundary conditions on the sphere surface.

3. MOTION OF THE SPHERE AT AN ARBITRARY ANGLE TO THE DIRECTION OF THE MAGNETIC FIELD VECTOR

Let the sphere move in the direction perpendicular to the vector $\mathbf{B}_0$ of an external magnetic field. We use, as before, a spherical coordinate system with the polar axis z passing through the sphere center in parallel to the vector $\mathbf{V}_0$ and the axis x along the magnetic field vector $\mathbf{B}_0$. The solution of Eqs. (1) will be sought in the form $b_r = f_r \cos\varphi$, $b_\theta = f_\theta \cos\varphi$ and $b_\varphi = f_\varphi \sin\varphi$, where φ is the azimuthal angle reckoned from the direction of the x axis. Then Eqs. (1) take the form

$$(\hat{T}_1 + \hat{T}_2\partial_\theta - \sin^{-2}\theta - 2)f_r - 2\hat{T}_2 f_\theta - 2f_\varphi \sin^{-1}\theta = 3K\cos\theta\sin\theta/r^2;$$
$$(\hat{T}_1 + \hat{T}_2\partial_\theta - 2\sin^{-2}\theta)f_\theta + 2\partial_\theta f_r - 2f_\varphi\cos\theta\sin^{-2}\theta = -K\cos 2\theta/r^2;$$
$$(\hat{T}_1 + \hat{T}_2\partial_\theta - 2\sin^{-2}\theta)f_\varphi - 2f_r\sin^{-1}\theta - 2f_\theta\cos\theta\sin^{-2}\theta = K\cos\theta/r^2. \tag{12}$$

The equation $\mathrm{div}\mathbf{b} = 0$ takes the form

$$r\partial_r f_r + 2f_r + \hat{T}_2 f_\theta + f_\varphi \sin^{-1}\theta = 0. \tag{13}$$

Dropping the expression $\hat{T}_2 f_\theta + f_\varphi\sin^{-1}\theta$ in Eqs. (12) and (13), we obtain an equation with respect to f_r. We seek the solution of this equation in the form $f_r = f_1(r)$. As a result, we obtain

$$r^4 f_1'' + 4r^3 f_1' - 4r^2 f_1 = 3K. \tag{14}$$

Since the right-hand sides of Eqs. (12) are expressed through associated Legendre functions, which depend on the argument $\cos\theta$, it is expedient to seek the solution of these equations in the form $f_\theta = a_0(r) + a_2(r)P_2^0(\cos\theta)$, $f_\varphi = a_1(r)P_1^0(\cos\theta)$, where a_0, a_1, and a_2 are unknown functions. Substituting these expressions into Eqs. (12) and (13) leads to the following system of equations:

$$r^2 a_0'' + 2ra_0' + 2a_1 + 3a_2 - 2f_1/3 = K/(3r^2), \qquad r^2 a_1'' + 2ra_1' - 2a_1 + 3a_2 - 2f_1 = K/r^2;$$
$$r^2 a_2'' + 2ra_2' - 6a_2 + 8f_1/3 = -4K/(3r^2), \qquad rf_1' + 2f_1 - 9a_2/2 = 0, \qquad a_0 + a_1 + a_2 = 0. \tag{15}$$

For a dielectric sphere, the normal component of the current density vector on the sphere surface vanishes. This condition is equivalent to the fact that the radial component of the magnetic perturbation curl is equal to zero at $r = R$, i. e.,

$$\partial_\theta(f_\varphi \sin\theta) + f_\theta = 0. \tag{16}$$

At the boundary of a dielectric sphere, all the components of the magnetic field perturbations should be continuous. The solution of Eqs. (14) and (15), which is bounded at zero and at infinity is sought with condition (16). From the linearity of the initial Maxwell equations it follows that with an arbitrary orientation of the sphere velocity vector with respect to the direction of an external magnetic field, the general solution of the problem is the sum of solutions obtained for the cases of longitudinal and transverse directions of the sphere motion. Denoting the projections of an external magnetic field on the x and z axes as $B_{0x} = B_0 \sin\beta$ and $B_{0z} = B_0\cos\beta$, respectively (here, β is the angle between the vectors $\mathbf{V}_0$ and $\mathbf{B}_0$), we write expressions for the components of the magnetic field perturbation accompanying the sphere motion at $r > R$:

$$b_r = \lambda B_0 \frac{R^2}{8r^2}\left(1 - \frac{3R^2}{5r^2}\right)[2\cos(\beta)\,(1 - 3\cos^2\theta) - 3\sin(\beta)\cos(\varphi)\sin(2\theta)];$$
$$b_\theta = \lambda B_0\frac{3R^4}{20r^4}[\cos(\beta)\sin(2\theta) - \sin(\beta)\cos(\varphi)\cos(2\theta)], \qquad b_\varphi = \lambda B_0 \frac{3R^4}{20r^4}\sin(\beta)\sin(\varphi)\cos(\theta). \tag{17}$$

Analysis of the obtained expressions shows that for far distances ($r \gg R$) the radial component b_r becomes much greater than the transverse components b_θ and b_φ. An asymptotic formula for the magnetic perturbations b_r is given by

$$b_r \approx \lambda B_0 \frac{R^2}{8r^2}[2\cos(\beta)\,(1-3\cos^2\theta) - 3\sin(\beta)\cos(\varphi)\sin(2\theta)].$$

Thus, the magnetic perturbation vector at far distances r decreases as r^{-2} and is directed to the moving sphere or oppositely. Analysis of expression (17) shows that b_r achieves the maximum values in the $y=0$ plane ($\varphi = 0$) at an angle $\theta_{\max} = (1/2)\arctan(B_{0x}/B_{0z}) = \beta/2$. Note that the magnetic dipole approximation, in which the perturbation amplitude depends on the distance as r^{-3}, is not applicable to this problem. This is due to the fact that in this problem the perturbation source, i. e., the velocity field in a liquid, is distributed in space, and therefore cannot be replaced by a point magnetic dipole. If the sphere center is at the origin of coordinates at the instant $t = 0$, then the dependence of the variables r and θ on the time in Eqs. (17) has the form $r = [x^2 + y^2 + (z - V_0 t)^2]^{1/2}$ and $\cos\theta = (z - V_0 t)/r$.

Passing to the laboratory reference frame, with respect to which the sphere moves with the velocity $\mathbf{V}_0$, requires transformations of the electromagnetic field. In the nonrelativistic case ($V_0 \ll c$), the formulas for magnetic perturbations (17) retain their form, but the coordinates r and θ depend on the time, and the electric field transforms in the following way: $\mathbf{E} = \bar{\mathbf{E}} + \mathbf{V}_0 \times \mathbf{B}_0$. The components of the electric field $\bar{\mathbf{E}}$ in the sphere-related coordinate system can be found using the Maxwell equation $\mathrm{div}\bar{\mathbf{b}} = \mu_0\sigma(\bar{\mathbf{E}} + \mathbf{V} \times \mathbf{B}_0)$. The components of the electric field $\mathbf{E}$ in the laboratory reference frame for $r > R$ have the form

$$E_r = V_0 B_0 \frac{R^3}{2r^3}\sin(\beta)\sin(\varphi)\sin(\theta), \qquad E_\theta = -V_0 B_0 \frac{R^3}{4r^3}\sin(\beta)\sin(\varphi)\cos(\theta);$$
$$E_\varphi = -V_0 B_0 \frac{R^3}{4r^3}\sin(\beta)\cos(\varphi). \tag{18}$$

In vector form, Eqs. (18) can be written as

$$\mathbf{E} = -\nabla\Phi, \qquad \Phi = \frac{\mathbf{r}(\mathbf{V}_0 \times \mathbf{B}_0)R^3}{4r^3}.$$

The electric field in the laboratory reference frame is of potential nature. The potential Φ of this field outside the sphere corresponds to the field of an effective dipole with the electric moment $\mathbf{d} = \pi\varepsilon_0(\mathbf{V}_0 \times \mathbf{B}_0)R^3$, where ε_0 is the electric constant. It follows that the electric perturbations can achieve the maximum values in the direction specified by the vector $\mathbf{V}_0 \times \mathbf{B}_0$, i. e., along the y axis. The electric field components are comparable in amplitude and decrease with the distance as r^{-3}. However, the amplitude of these perturbations, which is determined by the effective dipole moment $\mathbf{d}$, depends on the angle between $\mathbf{B}_0$ and $\mathbf{V}_0$ and, in particular, can vanish if these vectors are parallel.

4. DISCUSSION OF THE RESULTS

Figures 2 and 3 show the spatial distribution of the dimensionless radial perturbation of the magnetic field $b_r/(\lambda B_0)$ in the x, z plane, which is calculated by the first formula in Eqs. (17) using transformation of the coordinates $r = \sqrt{x^2 + z^2}$; $\theta = \arctan(x/z)$. For the parameters $\sigma = 5$ S/m, $V_0 = 5$ m/s, $B_0 = 5\cdot 10^{-5}$ T, and $R = 50$ m the obtained quantity λB_0 is equal to 75 nT. In this case, perturbation of the magnetic field near the sphere can achieve 10–15 nT. As is seen in Fig. 3, the polar diagrams of the distribution $b_r/(\lambda B_0)$ in the $y = 0$ plane have four lobes, whose location depends on the angle β between the vectors $\mathbf{V}_0$ and $\mathbf{B}_0$. The largest values of the magnetic perturbations are achieved in directions which make angles $\beta/2$ and $\pi + \beta/2$ with the vector $\mathbf{V}_0$. Two more local maxima are formed in the orthogonal directions.

The dependences of the radial component of the magnetic field perturbation on the distance for different angles β between the direction of the liquid flow and the magnetic field vector are presented in Fig. 4. Curves *1–4* are plotted for the polar angles $\theta = 0°$, $30°$, $60°$, and $90°$, respectively. At a dimensionless distance $r/R = 5$, the values b_r decrease by 1–2 orders of magnitude since this dependence is mainly determined by the relation $b_r \propto r^{-2}$. The different signs of the radial-perturbation projections at different angles θ are due to the property of closeness of the magnetic induction lines.

These results were obtained under the assumption of a small magnetic force acting on the conducting liquid, i. e., under the condition $|\mathbf{j} \times \mathbf{B}_0| \ll |\nabla P|$. The liquid pressure gradient near the sphere surface $|\nabla P| \approx \rho V_0^2/R$, where $\rho \approx 10^3$ kg/m^3 is the density of the liquid. Substituting the above-mentioned numerical values of the parameters into these relations, we obtain the following condition: $j \ll \rho V_0^2/(RB_0) = 10^7$ A/m^2, which is well fulfilled since the characteristic value of the current density in this problem $j = \sigma E \approx 3 \cdot 10^{-5}$ A/m^2.

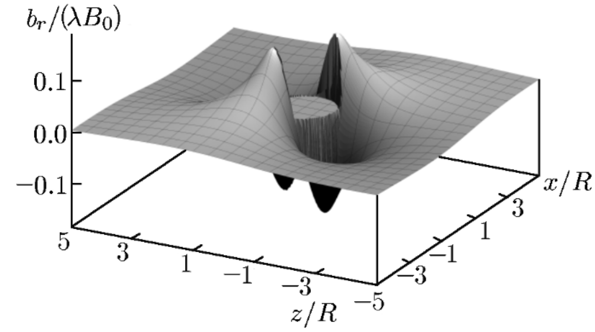

Fig. 2. Spatial distribution of the radial component of the dimensionless magnetic perturbation $b_r/(\lambda B_0)$ generated by a moving sphere in the $y = 0$ plane. The velocity vector $\mathbf{V}_0$ makes an angle $\beta = 60°$ with the vector $\mathbf{B}_0$.

In the laboratory coordinate system, the magnetic field perturbation at the measurement point depends on the time, since the coordinates are time dependent (see Eqs. (17)). For the dimensionless radial component of the magnetic perturbation at the x axis ($y = z = 0$) we have

$$b_r(\xi, \tau) = \frac{\lambda B_0}{4(\xi^2 + \tau^2)} \left[1 - \frac{3}{5(\xi^2 + \tau^2)}\right] \left[\cos(\beta)\left(1 - \frac{3\tau^2}{\xi^2 + \tau^2}\right) - 3\sin(\beta)\frac{\xi\tau}{\xi^2 + \tau^2}\right], \tag{19}$$

where $\xi = x/R$, $\tau = t/t_0$, and $t_0 = R/V_0$. Figure 5 shows the dependences of $b_r/(\lambda B_0)$ on the dimensionless

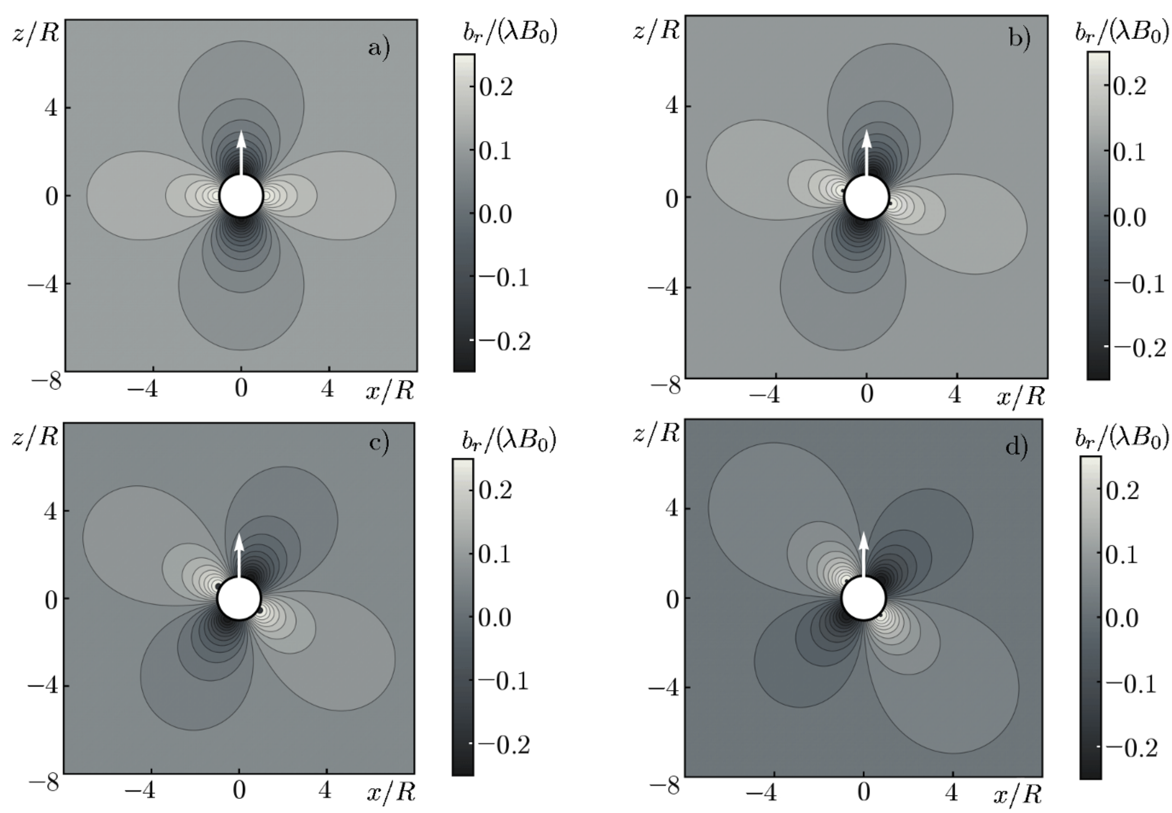

Fig. 3. Polar diagrams of the radial-component distribution of a dimensionless magnetic perturbation $b_r/(\lambda B_0)$ in the $y = 0$ plane for different inclination angles of the unperturbed magnetic field $\mathbf{B}_0$ with respect to the direction of the velocity $\mathbf{V}_0$ of the incident flow of liquid: $\beta = 0°$ (*a*), $30°$ (*b*), $60°$ (*c*), and $90°$ (*d*).

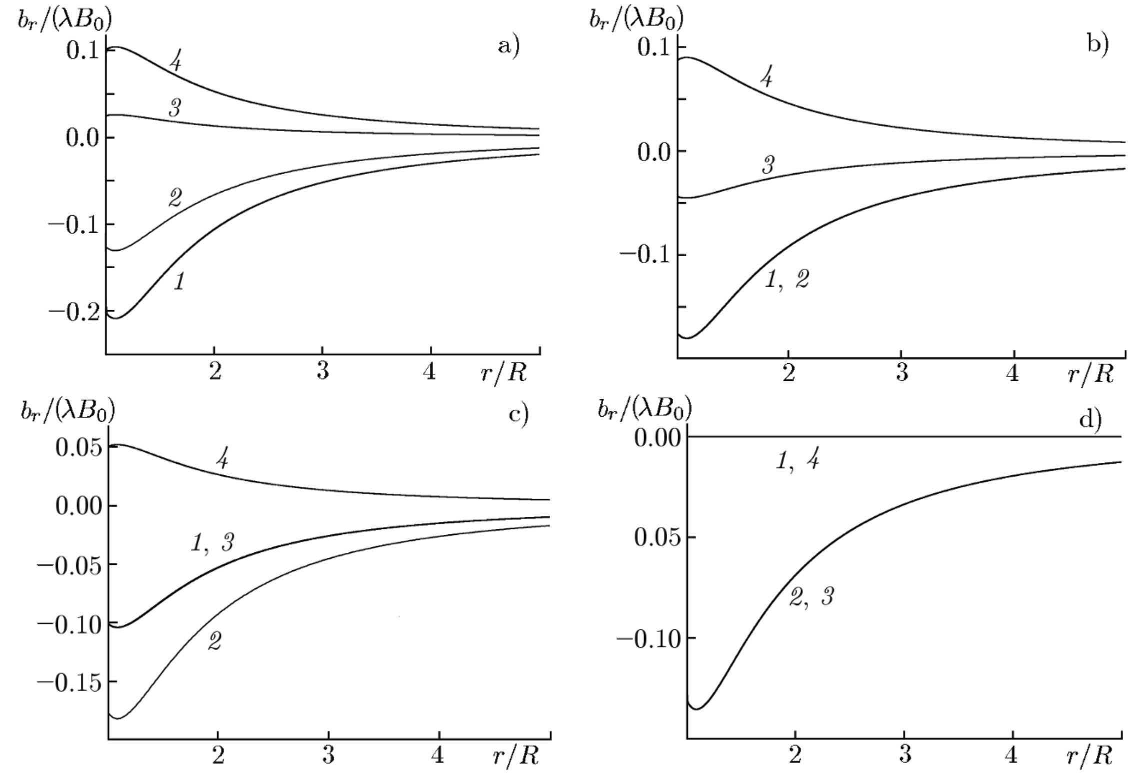

Fig. 4. Dimensionless dependences of the radial component of magnetic perturbation $b_r/(\lambda B_0)$ on the dimensionless distance r/R for different inclination angles β of the magnetic field with respect to the velocity direction of the incident flow of liquid. Panel *a* corresponds to the angle $\beta = 0°$, *b*, to $30°$, *c*, to $60°$, and *d*, to $90°$. Curves *1–4* correspond to the polar angles $\theta = 0°$, $30°$, $60°$, and $90°$, respectively.

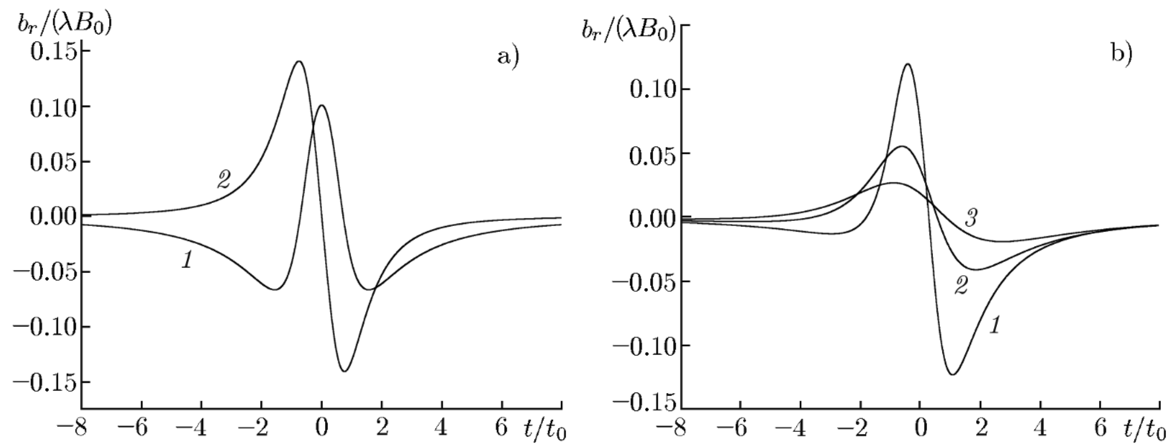

Fig. 5. Radial component of a dimensionless magnetic perturbation $b_r/(\lambda B_0)$ at the x axis in the laboratory coordinate system as a function of the dimensionless time t/t_0. On panel *a*, the coordinate $x = R$ and the angle $\beta = 0°$ (curve *1*) and $90°$ (curve *2*). On panel *b*, the angle $\beta = 45°$ and the coordinate $x = R$ (curve *1*), $2R$ (curve *2*), and $3R$ (curve *3*).

time t/t_0 at different distances from the sphere center and for different angles β between the vectors $\mathbf{V}_0$ and $\mathbf{B}_0$. The adopted values of the parameters yield $t_0 = 10$ s. The curves in Fig. 5*a* were plotted for $x = R$ and $\beta = 0°$ (curves *1*) and $90°$ (curve *2*). The curves in Fig. 5*b* were plotted for $\beta = 45°$ and $x = R$, $2R$, and $3R$ (curves *1–3*, respectively). It is seen in Fig. 5 that the dependence of the magnetic perturbation on the time is of alternating nature, and the signal may contain 2 or 3 peaks decreasing with the distance.

Equation (19) yields a frequency spectrum of the signal observed at the x axis. Performing a Fourier transform

$$\tilde{b}_r(\xi,\omega) = \int\limits_{-\infty}^{+\infty} b_r(\xi,\tau)\exp(i\omega\tau)\,\mathrm{d}\tau, \tag{20}$$

at large distances $r \gg R$ we find that

$$\tilde{b}_r(\xi,\omega) = (\pi\lambda B_0/8)\exp(-|\omega|\,t_0\xi)\left\{3\,|\omega|\,t_0\exp[i\beta\mathrm{sgn}(\omega)] - \frac{1}{\xi}\cos\beta\right\}. \tag{21}$$

Analysis of Eq. (21) shows that the frequency dependence of the modulus of the magnetic-perturbation spectrum is nonmonotonic, and the extremum arises at the frequencies $\omega_{\mp} = [3+2\cos\beta \mp (9+4\cos^2\beta)^{1/2}]/(6\xi t_0)$, where the minus sign corresponds to the minimum and the plus sign, to the maximum of the spectrum. Figure 6 shows the moduli of the magnetic-perturbation spectrum (21) as functions of the dimensionless frequency ωt_0 for the point with the coordinate $x = R$, which were calculated for different angles β. With the parameter values mentioned above, the maximum of the spectrum corresponds to the frequencies $\omega = 0.1$–0.2 Hz. It is seen in Fig. 6 that the frequencies of the maximum ω_+ and the minimum ω_- decrease with increasing angle β in accordance with the formula given above.

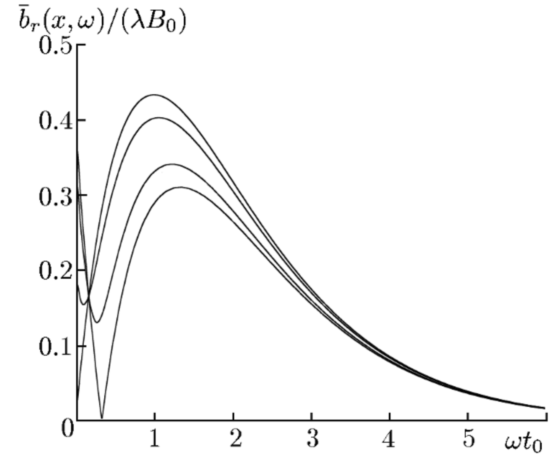

Fig. 6. The spectrum $\bar{b}_r(x,\omega)/(\lambda B_0)$ of the temporal dependence of the radial component of a dimensionless magnetic perturbation at the x axis in the laboratory coordinate system at the point $x = R$ for the angles $\beta = 0°$, $30°$, $60°$, and $90°$ (curves *1–4*, respectively).

Electric field (18) outside the sphere is distributed according to the dipole law with an effective dipole moment of the order of $7\cdot 10^{-9}$ C·m (for the above-mentioned parameters). The maximum of the field at the distance $r = 500$ m is estimated as 0.5 V/m. In the laboratory reference frame, the temporal dependence of an electric signal, as well as a magnetic one, is nonmonotonic, and the maximum of the signal spectrum lies in the same frequency range of the order of 0.1–0.2 Hz.

5. CONCLUSIONS

The obtained analytical solution of the problem makes it possible to estimate the magnetic perturbations caused by a moving solid dielectric body in the conducting liquid. If the body moves at a constant speed, and the flow of liquid is potential, then the magnetic perturbation amplitude appears to be proportional to the volume and speed of motion of the body. In an unbounded liquid, the magnetic perturbations decrease with the distance r as r^{-2} and the electric perturbations, as r^{-3}. Distribution of the electric field outside the sphere obeys the dipole law with an effective dipole moment, which is also proportional to the volume and speed of motion of the sphere. At far distances, the radial component B_r directed to or from the moving body dominates in the magnetic perturbations. The analysis has shown that the largest value of B_r is achieved in the plane in which the vectors of the unperturbed magnetic field $\mathbf{B}_0$ and the sphere velocity $\mathbf{V}_0$ lie. The polar diagram of the radial magnetic perturbation in this plane has the form of four orthogonal lobes, whose location depends on the angle β between the vectors $\mathbf{B}_0$ and $\mathbf{V}_0$. The magnetic perturbations are the maximum at angles $\beta/2$ and $\pi + \beta/2$ to the vector $\mathbf{V}_0$. Electric perturbations are the maximum in the direction perpendicular to the plane of the vectors $\mathbf{B}_0$ and $\mathbf{V}_0$. In the laboratory reference frame, the temporal dependences of the electric and magnetic signals are nonmonotonic. With the chosen parameters, the maxima of the spectra lie in the range of tenths of a hertz. Obviously, these conclusions remain valid for other boundary conditions for an electromagnetic field on the surface of a moving sphere, in particular, for a sphere made of conducting magnetic materials.

This work was supported by the Competitiveness Improvement Program of the National Research Nuclear University "Moscow Engineering Physics Institute."

REFERENCES

1. R. J. Urick, *Principles of Underwater Sound*, McGraw-Hill, New York (1983).
2. T. Stefanick, *Nonacoustic Detection of Submarines*, Scientific American, No. 3, p. 29. (1988)
3. O. Yaakobi, G. Zilman, and T. Miloh, *J. Eng. Math.*, **70**, Nos. 1–3, 17 (2011).
4. X.-J. Zhu and M.-Y. Xia, *Progress Electromagn. Res.*, **149**, 109 (2014).
5. J. T. Weaver, *J. Geophys. Res.*, **70**, No. 8, 1921 (1965).
6. C. S. Manoj, S. Maus, and A. Chulliat, *EOS Trans. AGU*, **92**, 13 (2011).
7. V. Surkov and M. Hayakawa, *Ultra and Extremely Low Frequency Electromagnetic Fields*, Springer, London, New York (2014).
8. L. D. Landau and E. M. Lifshitz, *Fluid Mechanics*, Vol. 6 of the Course of Theoretical Physics, Pergamon Press, Oxford (1987).